Collaborative Production, Strategic Action, and the Distribution of Income

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Abstract

We investigate strategic effect in collaborative production by multiple agents. For example, firms often jointly establish a new team for a project, or laborers compose a working team due to a certain requirement for the division of labor. An agent as a participator to the new team has nonnegligible influence to the performance of the project, and therefore is supposed to behave strategically. Along with game theoretic analysis of a certain model, we investigate the distribution of the income in the team, which proves to give a kind of rents to relatively low ability agents. And furthermore, we analyze the grouping decision by the entrepreneur and show that the member profile itself is distorted when such an entrepreneur exists.

JEL Classification Number : D20, D33, J23, J31, L23
Key words : Team production, distribution of income, division of labor, grouping decision

1.Introduction

Division of labor is still important theme in modern economics. To gain the benefit from it, various groups are composed by economic entities. Main production activities in our economy are practiced by such team production. We often see that firms jointly establish a new group and collaboratively produce goods. Laborers in manufacturing sector compose a working team and allocate their labor force for specific jobs.[1]

In such situations, each participator to the team has nonnegligible influence to the performance of the team production, and therefore is supposed to behave strategically. Here we analyze a certain model of the collaborative production with game theoretic methods. We show that the income distribution among the agents are not subject to Marginal Productivity Principle and indeed relatively low ability agents are enjoying rent. Furthermore, such distortion affects the composition of the team itself. Especially when the grouping is executed by a member of the team, even the second best situation[2] cannot be realized, due to a kind of free-ride incentive of the decision maker.

In this paper we analyze the case where the incentives of the agents are controlled by a certain incomplete contract. Basics of contract theory are summarized in Salanie (1997). Models of contract that deal multiple agents have already been discussed. For example, see Holmström and Milgrom (1991). Our model is completely deterministic, even though almost all incomplete contract literatures have stochastic factor in their model. In that sense, our paper is close to much older literatures, such as Holmström (1982).

On the other hand, our focus is heavily on the distribution of income. Literatures about the distribution of income are huge, among them, Matsui and Postlewaite (2000) is very much related with our study. In fact our result is similar to theirs in that they also showed a case where low ability workers receive a kind of rents. They analyzed the competitive market with a characteristic production function. We study a strategic situation and use rather common function, and we further investigate the effect of such distortion on the composition of the group.

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[1]For literature of the division of labor, see, for example, Kim(1989).

[2]The definition of the second best is introduced later. Briefly, it is the situation where the grouping is decided to maxmimize the joint profit of the team.

As the famous classic by Adam Smith maintained, the following two ideas are considered as the reasons why collaborative production is established. Technological requirement and the positive productivity effect from the division of labor. The basic model depends only on the former, that is, the collaboration occurs because the positive production is impossible without it. The aspect of specific effect of the team production is visited in the section for numerical analysis.

The nonnegligible influences of the agents imply that such potential entrants for the new team are limited in number. Usually such limitation is due to some technological constraints. The project requires agents specific technologies to practice the jobs of the project. Such technologies are often partially embodied in the persons and/or the firms, therefore nothing to say that learning them is highly costful, even observing and verifying how they work is difficult. The inverifiability is the cause of the incompleteness of contract in our model.

The rest of the paper is constructed as follows. Section 2 introduces the basic model to analyze. Section 3 analyzes equilibria and shows a qualitative result. Section 4 investigates the grouping decision by numerical method. Section 5 shortly concludes.

2.Model

Let us start with the production function

$$F(x) = g \prod_{i=1}^{M} x_i^{\alpha_i}.$$

This function has the Cobb=Douglas form. x is the vector of contributed efforts by the agents, which is $x = (x_1, \ldots, x_M)$. M is the number of the agents required for the project, which is given as a parameter, and we assume $M \geq 2$. α_i is the relative ability of agent i, and we set $\sum_{i=1}^{M} \alpha_i = 1$. Multiplier g is the coefficient which represents the team production effect. The team production effect is explained and specified later, and as for now it is dealed as a fixed positive parameter.

There are infinite numbers of agents in the economy. Each agent can be hired as a member of the project. They have their own positive ability parameters. We denote the ability of agent j as a_j. When a team of M members is established, depending on the members, the vector of relative abilities is determined as follows:

$$\alpha = (\alpha_1, \ldots, \alpha_M),$$
$$\text{where} \quad \alpha_i = \frac{a_i}{\sum_{j=1}^{M} a_j}.$$

Assumption

For tractability, we simplify the profile of the agents' abilities. There are only two types of the ability, a_H *and* a_L $(a_H > a_L)$.

We assume that there are sufficiently many agents of both types so that any type profile of the members of the team is possible. (In fact, if we assume only that there are at least M agents of the type a_H and M agents of the type a_L, it is enough.) With this simplification, the relative abilities of the members also are of two types. We denote the relative ability of the member with type a_H(resp. a_L) as α_H(resp. α_L).

Effort of each agent increases the production, but it inevitably costs. The individual cost function is:

$$C_i(x_i) = x_i^{\sigma} \quad (\sigma \geq 2).$$

Now we set up the game. It is a dynamic game with four stages, numbered 0 to 3. Players are firms or laborers, which we call agents here, and the chance move.

At stage 0, among agents, one agent is randomly chosen as entrepreneur. The entrepreneur has the project which requires the participation by M agents to practice itself.

At stage 1, the entrepreneur hires the agents. Agent who is offered a job decides whether to accept or reject. The agent is assumed to accept the job when her profit (defined below) is nonnegative.

At stage 2, agents' share of future income from the project is determined. The share is expressed by a simplex vector θ with dimension of M, $\theta = (\theta_1, \ldots, \theta_M)$. We assume that θ is determined to maximize joint profit.

At stage 3, agents, including the entrepreneur, decide how much effort to put into the project. Agents act to maximize their own profit. Agent's individual profit is defined as below.

$$\theta_i F(x) - C_i \quad (i = 1, \ldots, M).$$

With simple summation, the joint profit is given by

$$F(x) - \sum_{i=1}^{M} C_i.$$

Maximization of the joint profit by θ at the stage 2 seems to be somewhat controversial. Literatures of contract theory tell us that the sharing rule can be optimal with respect to profit maximization if it is allowed to dispose a part of the income or to depend on the effort levels of the agents. However, we regard such contract as being irrelevant to the situation we are going to study here.

The effort exhibited by an agent is often inverifiable, even if it is observable, therefore individual cost is also not verifiable. Thus any contract which depends on the effort level profile or cost profile seems not realistic.[3]

Furthermore, the maximization of the joint profit implies an implicit assumption about bargaining power of the agents. The agents are symmetric with respect to the bargaining power over the income. In other words, even the entrepreneur does not have any surplus bargaining power against the others. The project is owned by the entrepreneur, however, it is never accomplished without the participation by other $M - 1$ members. So we think that the entrepreneur cannot assert any advantage over the residual.

3.Equilibrium

We solve for Subgame Perfect Nash Equilibria using the backward induction method. Beforehand of the analysis, we are going to exclude trivial cases.

Lemma 1

(1) There exist many equilibria where there are at least i and j in $\{1, \ldots, M\}$, $i \neq j$, $x_i = x_j = 0$. θ can be arbitrary anything.

(2)There exists an equilibrium where θ is strictly greater than zero and the joint profit is positive, in other words, there is no $k \in \{1, \ldots, M\}$ such that $\theta_k = 0$ and $\pi = 0$.

[3] Even if the contract is prohibited to use the effort levels as information, the optimal outcome can be enforced as long as free disposal of income is assumed (see Holmström(1982)). However, the free disposal assumption is still not realistic. After the realization of income, do the members really agree with abandoning a part of the harvest ? Considering their close relationships as collaborators in a team, they will get into the negotiation over the residual. Regarding the re-negotiation possibility, the contract must allocate all of the income to the members.

Proof: See Appendix 1.

Afterward, we focus on only relevant equilibria, which have strictly positive θ and x's.

First Best

As a benchmark, we first solve for the first best, which is the outcome when all x's are controlled by one decision maker to maximize the joint profit. The problem is shown by

$$\max_{x}\left[g\prod_{k=1}^{M}x_k^{\alpha_k}-\sum_{k=1}^{M}x_k^{\sigma}\right].$$

From necessary first order condition, we can derive the first best x's, denoted x^*'s just below. (See Appendix 2)

$$\forall i\in\{1,...,M\},\ x_i^*=\left(\prod_{k=1}^{M}(\alpha_k)^{\frac{\alpha_k}{\sigma}}\cdot\frac{\alpha_i^{\frac{\sigma-1}{\sigma}}g}{\sigma}\right)^{\frac{1}{\sigma-1}}.$$

Then, First Best production output is derived as follows. (Also see Appendix 2)

$$F=g\prod_{k=1}^{M}x_k^{\alpha_k}=\left(\frac{g^{\sigma}}{\sigma}\prod_{k=1}^{M}\alpha_k^{\alpha_k}\right)^{\frac{1}{\sigma-1}}.$$

Stage 3

At stage 3, α and θ is given and arbitrary agent i decides her effort level x_i. The individual problem is

$$\max_{x_i}\left[\theta_i g\prod_{k=1}^{M}x_k^{\alpha_k}-x_i^{\sigma}\right].$$

Again using NFOC, we can derive equilibrium effort levels (See Appendix 3a).

$$\forall i\in\{1,...,M\},\ x_i=\left[\prod_{k=1}^{M}(\alpha_k\theta_k)^{\frac{\alpha_k}{\sigma}}\cdot\frac{(\alpha_i\theta_i)^{\frac{\sigma-1}{\sigma}}g}{\sigma}\right]^{\frac{1}{\sigma-1}}.$$

Clearly we can see that x_i is strictly less than x_i^*.[4]

Stage 2

Now we consider the joint profit maximization by the control of θ. By substitution, we have the problem as below.

$$\max_{\theta}\left[\frac{g^{\sigma}}{\sigma}\prod_{k=1}^{M}(\alpha_k\theta_k)^{\alpha_k}\right]^{\frac{1}{\sigma-1}}-\sum_{k=1}^{M}\left[\prod_{j=1}^{M}(\alpha_j\theta_j)^{\alpha_j}\cdot\frac{(\alpha_k\theta_k)^{\sigma-1}g^{\sigma}}{\sigma^{\sigma}}\right]^{\frac{1}{\sigma-1}}\quad \text{s.t.}\quad \sum_{k=1}^{M}\theta_k=1.$$

With some steps, we have an implicit equation of θ (See Appendix 3b).

$$\forall i,j\in\{1,...,M\},\ \frac{\alpha_i}{\theta_i}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{(\sigma-1)\alpha_i}{\sigma}=\frac{\alpha_j}{\theta_j}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{(\sigma-1)\alpha_j}{\sigma}.$$

[4] This distortion effect in the team production situation is already generically shown by Holmström(1982).

Beforehand solving the equation explicitly, implicit representation above tells us some characteristics of the second best distribution of the income.

Proposition 1

(1) $\forall i, j \in \{1, ..., M\}$,
$\alpha_i = \alpha_j \Leftrightarrow \theta_i = \theta_j$, $\alpha_i > \alpha_j \Rightarrow \frac{\alpha_i}{\theta_i} > \frac{\alpha_j}{\theta_j}$, *and* $\alpha_i > \alpha_j \Rightarrow \theta_i > \theta_j$.

(2) $\exists\, l \in \{1, ..., M\}$, $\forall i \in \{1, ..., M\}$,
$\alpha_l < \alpha_i \Rightarrow \alpha_i > \theta_i > \sum_{k=1}^{M} \alpha_k \theta_k$,
$\alpha_l > \alpha_i \Rightarrow \alpha_i < \theta_i < \sum_{k=1}^{M} \alpha_k \theta_k$,
$(\alpha_l = \alpha_i \Rightarrow \alpha_i \geq \theta_i \geq \sum_{k=1}^{M} \alpha_k \theta_k)$.

Proof: See Appendix 4.

Let us explain the implications of the proposition. The first component of the proposition says that owners of the same productivity receive the same amount of income, and the agent-wise revenue is increasing function of its productivity. The second component says, however, that the payment rule is not subject to Marginal Production Principle and the agents with relatively low productivity receive rents from those with relatively high productivity. In other words, agents with relatively high productivity are sacrificed for the others.

Such rent bias is due to the bottleneck property of the production technology. Concavity of the production function requires that the effort levels are smoothed so that any of them does not deter the production activity. When agents act strategically, the project must give sufficient incentives to all of the agents, including those with low productivities. Then, those with high abilities are sacrificed for allocation of appropriate incentives.

More intuitively, the division of labor naturally makes the members specialize into their particular allocated jobs. In that case, lazy work by one of them inevitably deters the performance of the whole project. Therefore, team production is likely to have bottleneck property over the members' efforts. Thus, any sabotage must be avoided, as long as the project pays. To encourage all the members, an ability-depending allocation system is inappropriate, and the income distribution tends to be more equalized.

The threshold that discriminates sacrifices and rent-takers can be said, in a sense, the average productivity of the project. Interpreting α_i as the agent i's ability, θ_i is an incentive given to the agent i by the contract. Therefore $\sum_{k=1}^{M} \alpha_k \theta_k$ can be said to be practically induced value of productivity of the project. Those with higher productivity than the average are contributing sacrificially to the project.

Agents' dividends are equalized than the case of the First Best, and it arises from the fact that the total output is divided by the contract and it controls the incentives of the agents acting strategically. The result itself is very similar to that of Matsui and Postlewaite(2000), however their key point is that even in a competitive situation, under a certain production function, the distortionary allocation can be analytically proved. Our focus is especially on the team production situation, and using popular and simple functions, we are doing detailed study of the situation. Particularly, we analyze the effect of the distortion on grouping of the team and its efficiency.

Given α, we can solve for θ explicitly and it proves to be the unique solution (for the proof, see Appendix 5). That is:

$$\theta_H = \frac{B - \sqrt{D}}{A},$$

$$\text{where} \quad A = 2\sigma M m \alpha_H - 2\sigma m,$$
$$B = \sigma M - \sigma m - \sigma + M\alpha_H + Mm\alpha_H^2,$$

$$D = (\sigma M - \sigma m - \sigma + M\alpha_H + Mm\alpha_H^2)^2$$
$$-4(\sigma Mm\alpha_H - \sigma m)[(\sigma M - \sigma m - 1)\alpha_H + m\alpha_H^2]$$
$$\theta_L = \frac{1 - m\theta_H}{M - m},$$
m: the number of the members with type of a_H.

We have not solved the Stage 1 analytically yet. However, due to the complexity of the value of θ, analytical study is difficult. Therefore we shift to the numerical analysis.

4.Numerical Analysis

At the stage 1, the type profile of the members is determined. This can be simply expressed by the number of the high type members, m. Thus the entrepreneur maximizes her own profit by the control of m. We study this process by numerical computation.

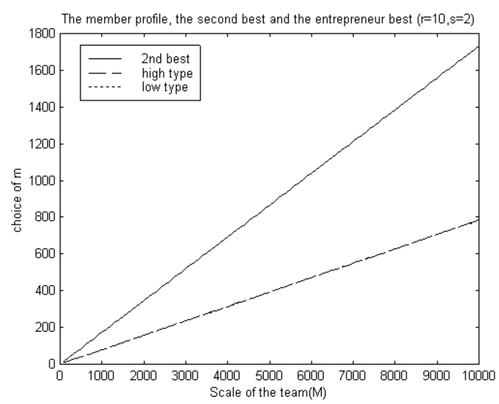
We focus on three kinds of maximization. First, the maximization of total profit as the second best of the model. Here, given the equilibrium actions of the agents, the total profit is maximized by the control of m. Second, the maximization of the individual profit of the high type agent, in other words, the entrepreneur's maximization when she is of the high type. Third, respectively, the maximization of the individual profit of the low type agent, which is the case that the entrepreneur is of the low type.

Because the equilibrium effort functions x and dividends θ have been explicitly solved, we can compute the maximization problem over m, if we fix some exogenous parameters. Here we set $\sigma = 2, M = 100$ and $r \equiv a_H/a_L = (\alpha_H/\alpha_L =)10$. Unless especially notified, these parameters are unchanged. We note that there occurred no qualitative change in the results when we modified the parameters.

Basic case

We assume that g in the production function is a constant. We name this case as the basic case, because afterward in this section, g is changed into some kinds of nondegenerate function of m.

In the basic case, the second best m and the individual bests differ in a characteristic way. Below, the horizontal axis stands for M and the vertical axis stands for m.



As seen above, the second best m is always derived as an fixed proportion of the team, for arbitrary M. The individual best m also shows a proportional characteristics, never depending on what type she is of. In the case above($r = 10,\ \sigma = 2$), the second best is about 17% of the team and the entrepreneur best is about 8% of the team. Thus, when an entrepreneur decides on the ability profile of the members, whatever her own type is, the determined profile is always much lower the social optimal. As we can see in an appendix later, this relationship between the second best and the entrepreneur best is highly robust.

This result is intuitively explained by the notion of the free ride. For each member, the others' effort cost is external. They tend to depend on the others' efforts and make their own effort insufficient from the viewpoint of the social best. The free ride takes the economy into the second best. Furthermore, this free ride incentive affects the grouping decision. Given the strategic situation, the second best member profile is too costful for each agent, because it requires relatively much effort. Inviting fewer high type agents and giving each of them strong incentive to make an effort, the low type agent (therefore low type entrepreneur) can gain from the free ride. On the other hand, the high type agent (therefore the high type entrepreneur) can gain from higher share over the profit. [5]

The gap between the second best and the individual preference is partly due to the lack of the team production effect in the production function. As empirically and routinely argued, the member profile itself may affect the productivity. Afterward, we introduce such team production effect, more practically, modify the parameter g into a function of m.

Team production effect

Because x and θ are independent from g, we do not have to solve for them again and can just introduce new g and compute the solutions of m. In this subsection, we introduce three cases of the modification of g.

First, a case where the proportion of the high type members in the team positively affects the productivity. With more high type agents, the work as a whole may be done more efficiently. g is set to be the (normalized) average of the agents' abilities here. Second, a case where the best ratio of the high type and the low type agents is given. When the agents have to be matched and make small work groups with a certain ratio of the high type and the low type agents, the most desirable m and $M-m$ for the maximization of g is uniquely determined, so that $m/(M-m)$ should be the ratio. Any violation of the ratio exhibits some inefficiency. We set g as a certain single-peaked convex function of m. And third, a case where the dispersion of the abilities of the agents harms the efficiency. On the other side of the individual abilities, it is said that the agents' abilities are more diverse, the whole team works better, because the higher types can train and inspire the lower types. This effect is studied and asserted by Hamilton, Nickerson, and Owan(2003). Here we take the standard deviation of the agents' abilities and make g positively correlated with that.

Since the results of the basic case exhibits a kind of stability with respect to the proportion in the team, afterward we focus on that proportion, i.e. m/M.

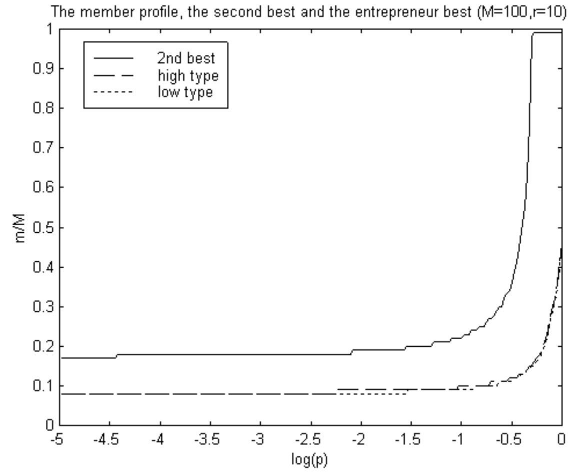
Case 1

We specify the g as follows:

$$g = (1-p) + p\left\{\frac{m(r-1)+M}{rM}\right\} \qquad \text{where } r = \frac{a_H}{a_L}.$$

Where p is arbitrary. the typical case is when p is 1. The larger p is, the more strongly the team production effects appear. We simulated on various p and studied how this setting differs from the basic case.

[5] Taking $\alpha_H/\alpha_L = r$ as given, $\alpha_H = \frac{r}{mr+(M-m)}$. It can be easily checked that α_H is decreasing function of m.



In the figure above, the horizontal axis represents natural logarithm of p and the vertical axis is for m, of which the unit is a person. This scaling is not changed afterward.

We can see that, as the effect of the averageous ability is enhanced(i.e. p increases), the second best m responds earlier than the entrepreneur bests and rises up. The second best m is larger than the individual best, whatever her type is. For the entrepreneur as an individual, there is a trade-off between the free ride and the positive team production effect. Therefore the response of the entrepreneur delays for that of the second best.

Furthermore we can note that, considering that their coincidence is at the corner, the second best m leaves farther from the individual preferred m as the averageous ability affects more. This property is also robust against the shift of the parameters. The averageous ability is maximized when all of the members are of the high type. At the extreme, the second best is realized at the corner solution. However for the entrepreneur as an individual, the corner solution is much costful and the benefit from the team production effect is discounted by her share. This logic, of course, is weakened when the difference between the high type and the low type is relatively small. In fact, when we set $r = 2$, the second best and the entrepreneur best may coincide at the corner.[6]

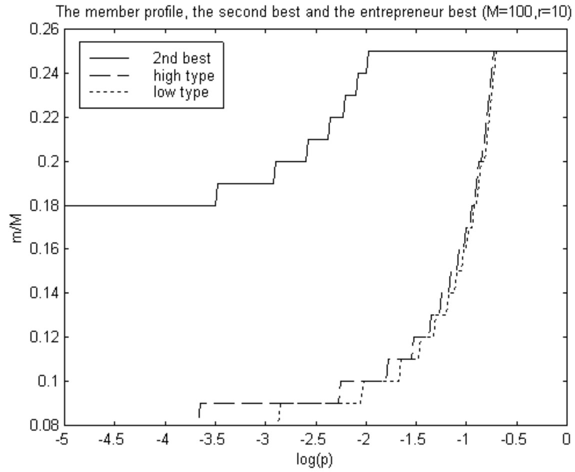
Case 2

As a single-peaked function referred above, we take:

$$g = \begin{cases} (1-p) + p\left(\frac{m-\frac{M}{4}}{M/4}\right) & \left(m \leq \frac{M}{4}\right) \\ (1-p) + p\left(\frac{\frac{M}{4}-m}{3M/4}\right) & \left(m > \frac{M}{4}\right) \end{cases} .$$

Again p is arbitrary. This function has an unique peak when $m = M/4$ and has the same value $(1-p)$ on both of its ends. It is easily expected that the chosen m's will get closer to the peak as p becomes larger.

[6] As in Appendix 5, there are some cases where the high type entrepreneur's best exceeds the second best, however we consider them as the results of computational discretization. Indeed such passover occurs only when the high type entrepreneur's best reaches the corner, the second best is next to the corner(i.e. $M-1$) and the scale of the team is relatively small(i.e. $M \leq 100$).



When p is small, the function g is almost flat and there is no difference from the basic case. As p increases, the unique peak of the function g affects and both of the second best and the individual preference converge to the point. This time the second best reaches the point earlier again,[7] however the response of the individual preference is quick. The entrepreneur compares the gain from the free ride and that from the team production, and therefore delays the adaptation to the effect of the team production function, however once the gain from the free ride is overwhelmed, she does not hesitate to take the advantage.

Here it can be seen that the high type entrepreneur's decision sometimes differs from that of the low type. Though it is small gap in the team, it implies the type of the entrepreneur may affect the performance of the team. And as we know, the low type entrepreneur's decision never exceeds that of the high type. The relationship is not limited in the case 1. Whenever the gap prevails, the high type preferres larger m than the low type.

Case 3

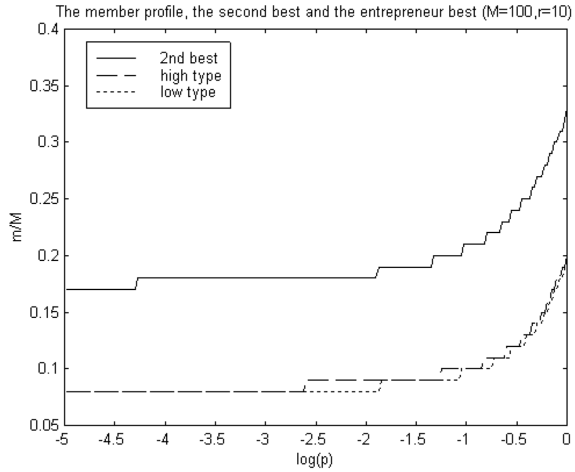
Now we take the standard deviation of the agents' abilities and use it as a parameter.

$$g = (1-p) + p\left(\sqrt{\frac{m(1-\mathrm{AVR})^2 + (M-m)\left(\frac{1}{r} - \mathrm{AVR}\right)^2}{M-1}}\right),$$

$$\text{where} \qquad \mathrm{AVR} = \frac{mr + (M-m)}{rM}, \quad r = \frac{a_H}{a_L}.$$

Still p is arbitrary, though in this case g is not standardized in the sense that the maximum value of g is not necessarily 1. However qualitative result does not change if we standardize g. The averageous ability above(AVR) and the whole g are calculated under the standardization which sets $a_H = 1$ and $a_L = 1/r$.

[7]Even if the point is between their starting points, this does not change.



Here, which is earlier the response of the second best or that of the individual preference cannot be distinguished. Indeed, it depends on the parameters(more practically, M and r). The second best m is greater than that of the individual preference, however we can tell that their gap is shrinking as p increases. Note that in the case 1, the gap tends rather to expand. This is due to the property of the standard deviation. At the corner, where m is 0 or M, the standard deviation is of course 0. Therefore the maximization of the standard deviation is done at a certain inner solution, in contrast to the case of the averageous ability. So both of the second best and the entrepreneur bests are induced to get closer.

With the same logic of that in the case 1, this property is weakened when r becomes larger, because the maximizer of the standard deviation gets closer to the upper corner of m. When the maximizer is almost equal to M, the gap is relatively large again. This implies that when the variance-oriented production effect is significant, or such working environment is established, the gap between the second best and the third best, if we name the entrepreneur's choice such, can move whichever direction, wider or thinner.[8]

More on the basic case

Let us introduce some facts which are found in the study of the basic case.

Shift of Second Best m

r \M	10	100	1000	10000
2	4	39	386	3863
4	3	28	283	2828
6	2	23	230	2300
8	2	20	197	1967
10	2	17	173	1732
20	1	11	113	1133

With fixed scale and moving r, it is found that the second best m is monotonely decreasing to r. On the other hand, the entrepreneur's bests' shifts are not monotone. There exist inner ranges of r which maximize the entrepreneur's choice, for each of the type of the entrepreneur. They are as shown below.

[8]See Appendix 6 for comparison of the different cases.

Maximizers of m

Type \M	10	100	1000	10000
H	all($m = 1$)	4.2-9.0($m = 9$)	5.3-6.6($m = 87$)	5.7-6.1($m = 867$)
L	all($m = 1$)	4.7-7.7($m = 9$)	5.5-6.4($m = 87$)	5.8-6.1($m = 867$)

Such peak of the choice is stable with respect to the proportion in the team. And the range of r which gives the peak is also stable across the cases.

5.Conclusion

In this paper we analyzed a certain kind of team production activity with strategic behavior of agents.

When multiple agents practice the collaborative production activity, they act strategically and maximize their own private profit. Along with a certain incompleteness of the contract, this strategic effect distorts the distribution of income and agents who have relatively low productivity enjoy rent. The distortion even sheds the shadow over the group construction.

By numerical analysis, the distortion in the group composition is studied. The proportion of the high ability agents in the team tends to be determined by the ratio of the higher ability over the lower ability. With respect to the entrepreneur's type, the high type entrepreneur's choice of the number of the high type agents is greater or at least equal to that of the low type entrepreneur.

Here we simplified the bargaining process of the share determination and neglected possible advantage the entrepreneur may have in the bargaining process. It is dedicated to the future research.

Appendix 1

Proof of Lemma 1:

(1) Given that an agent contributes nothing, other agent's best response is unique and also contributing nothing. So two agents contributing nothing is sufficient for autarky. In this case θ does nothing to income distribution, therefore any θ can be equilibrium outcome.

(2) Clearly from agents' private profit function, at the second stage, if θ_k is equal to zero, optimal strategy for agent k is unique and $x_k = 0$. And that means total output is zero. With strictly positive θ, for any i there exists strictly positive agent i's best response, given that the others' contributions are all positive. So in an equilibrium outcome, due to the concavity of agent's optimization problem and the fact that $\forall i,\ C_i'(0) = 0$, every private profit, and therefore also the joint profit, can be strictly positive. And in that equilibrium, π is strictly greater than zero. Q.E.D.

Appendix 2

First Best derivation:

Necessary first order condition proves to be

$$\forall i \in \{1, ..., M\},\ \alpha_i g \prod_{k=1}^{M} x_k^{\alpha_k} \cdot x_i^{-\sigma} - \sigma = 0.$$

This is satisfied for any i. Hence it is clear that

$$\forall i, k \in \{1, ..., M\},\ \left(\frac{x_k}{x_i}\right)^{\sigma} = \frac{\alpha_k}{\alpha_i} \Rightarrow x_k = \left(\frac{\alpha_k}{\alpha_i}\right)^{\frac{1}{\sigma}} x_i.$$

Substituting it into previous first order condition, we can derive the first best x's, denoted x^*'s just below.

$$\forall i \in \{1, ..., M\},\ x_i^* = \left(\prod_{k=1}^{M} (\alpha_k)^{\frac{\alpha_k}{\sigma}} \cdot \frac{\alpha_i^{\frac{\sigma-1}{\sigma}} g}{\sigma}\right)^{\frac{1}{\sigma-1}}.$$

First Best production output is derived in the following way.

$$\begin{aligned}
x_i^{*\alpha_i} &= \left(\prod_{k=1}^{M} (\alpha_k)^{\frac{\alpha_k \alpha_i}{\sigma}} \cdot \left(\frac{\alpha_i^{\frac{\sigma-1}{\sigma}} g}{\sigma}\right)^{\alpha_i}\right)^{\frac{1}{\sigma-1}}, \\
\Rightarrow \prod_{i=1}^{M} x_i^{*\alpha_i} &= \left(\prod_{k=1}^{M} \alpha_k^{\frac{\alpha_k}{\sigma}} \prod_{i=1}^{M} \alpha_i^{\frac{(\sigma-1)\alpha_i}{\sigma}} \cdot \frac{g}{\sigma}\right)^{\frac{1}{\sigma-1}} = \left(\prod_{k=1}^{M} \alpha_k^{\alpha_k} \cdot \frac{g}{\sigma}\right)^{\frac{1}{\sigma-1}}, \\
\Rightarrow F = g \prod_{k=1}^{M} x_k^{*\alpha_k} &= \left(\frac{g^{\sigma}}{\sigma} \prod_{k=1}^{M} \alpha_k^{\alpha_k}\right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

Appendix 3

(a) *Derivation of the equilibrium effort levels and the output. (given θ)*

$$\max_{x_i} \left[\theta_i g \prod_{k=1}^{M} x_k^{\alpha_k} - x_i^{\sigma}\right].$$

Necessary first order condition proves to be

$$\forall i \in \{1, ..., M\},\ \alpha_i \theta_i g \prod_{k=1}^{M} x_k^{\alpha_k} \cdot x_i^{-\sigma} - \sigma = 0.$$

This is satisfied for any i, so it is clear that

$$\forall i,k \in \{1,...,M\},\ \left(\frac{x_k}{x_i}\right)^{\sigma} = \frac{\alpha_k\theta_k}{\alpha_i\theta_i} \Rightarrow x_k = \left(\frac{\alpha_k\theta_k}{\alpha_i\theta_i}\right)^{\frac{1}{\sigma}} x_i.$$

Substituting it into previous first order condition, we can derive the equilibrium x's, denoted x^*'s just below.

$$\forall i \in \{1,...,M\},\ x_i^* = \left[\prod_{k=1}^{M}(\alpha_k\theta_k)^{\frac{\alpha_k}{\sigma}} \cdot \frac{(\alpha_i\theta_i)^{\frac{\sigma-1}{\sigma}} g}{\sigma}\right]^{\frac{1}{\sigma-1}}.$$

Equilibrium production output is derived in the following way.

$$\begin{aligned}
x_i^{*\alpha_i} &= \left[\prod_{k=1}^{M}(\alpha_k\theta_k)^{\frac{\alpha_k\alpha_i}{\sigma}} \cdot \left(\frac{(\alpha_i\theta_i)^{\frac{\sigma-1}{\sigma}} g}{\sigma}\right)^{\alpha_i}\right]^{\frac{1}{\sigma-1}}, \\
\Rightarrow \quad \prod_{i=1}^{M} x_i^{*\alpha_i} &= \left[\prod_{k=1}^{M}(\alpha_k\theta_k)^{\frac{\alpha_k}{\sigma}} \prod_{i=1}^{M}(\alpha_i\theta_i)^{\frac{(\sigma-1)\alpha_i}{\sigma}} \cdot \frac{g}{\sigma}\right]^{\frac{1}{\sigma-1}} = \left[\prod_{k=1}^{M}(\alpha_k\theta_k)^{\alpha_k} \cdot \frac{g}{\sigma}\right]^{\frac{1}{\sigma-1}}, \\
\Rightarrow \quad F = g\prod_{k=1}^{M} x_k^{\alpha_k} &= \left[\frac{g^{\sigma}}{\sigma}\prod_{k=1}^{M}(\alpha_k\theta_k)^{\alpha_k}\right]^{\frac{1}{\sigma-1}}.
\end{aligned}$$

(b) *Derivation of the equation about θ*

$$\max_{\theta} \left[\frac{g^{\sigma}}{\sigma}\prod_{k=1}^{M}(\alpha_k\theta_k)^{\alpha_k}\right]^{\frac{1}{\sigma-1}} - \sum_{k=1}^{M}\left[\prod_{j=1}^{M}(\alpha_j\theta_j)^{\alpha_j} \cdot \frac{(\alpha_k\theta_k)^{\sigma-1} g^{\sigma}}{\sigma^{\sigma}}\right]^{\frac{1}{\sigma-1}} \quad \text{s.t.} \quad \sum_{k=1}^{M}\theta_k = 1.$$

Noting that $\frac{g^{\sigma}}{\sigma}$ and $\prod_{k=1}^{M}\alpha_k^{\alpha_k}$ are just constants, the problem can be simplified.

$$\max_{\theta}\prod_{k=1}^{M}\theta_k^{\frac{\alpha_k}{\sigma-1}}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right) \quad \text{s.t.} \quad \sum_{k=1}^{M}\theta_k = 1.$$

The first order condition over θ_i is

$$\frac{\alpha_i}{\sigma-1}\prod_{k=1}^{M}\theta_k^{\frac{\alpha_k}{\sigma-1}}\left\{\theta_i^{-1} - \theta_i^{-1}\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma} - \frac{\sigma-1}{\sigma}\right\} = \lambda,$$

where λ is a Lagrange multiplier. Modifying this equation

$$\frac{\alpha_i}{\theta_i}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right) - \frac{(\sigma-1)\alpha_i}{\sigma} = \frac{(\sigma-1)\lambda}{\prod_{k=1}^{M}\theta_k^{\frac{\alpha_k}{\sigma-1}}}.$$

Since i is arbitrary, we have

$$\forall i,j \in \{1,...,M\},\ \frac{\alpha_i}{\theta_i}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right) - \frac{(\sigma-1)\alpha_i}{\sigma} = \frac{\alpha_j}{\theta_j}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right) - \frac{(\sigma-1)\alpha_j}{\sigma}.$$

Appendix 4

Proof of Proposition 1:

(1) Note that $\sum_{k=1}^{M}\alpha_k\theta_k < 1$, so that $(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}) > \frac{\sigma-1}{\sigma}$. $\alpha_i = \alpha_j \Rightarrow \theta_i = \theta_j$ and $\alpha_i > \alpha_j \Rightarrow \frac{\alpha_i}{\theta_i} > \frac{\alpha_j}{\theta_j}$ are obvious. The condition above is rewritten as follows.

$$\alpha_i\left\{\frac{1}{\theta_i}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{\sigma-1}{\sigma}\right\} = \alpha_j\left\{\frac{1}{\theta_j}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{\sigma-1}{\sigma}\right\},$$

where $\left\{\frac{1}{\theta_i}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{\sigma-1}{\sigma}\right\} > 0$. Therefore, if $\alpha_i > \alpha_j$, $\theta_i > \theta_j$ is necessary. The same equation tells that if θ_i is equal to θ_j, α_i and α_j must be equal.

(2) Multiplying $\theta_i\theta_j$ on both sides of the equation, we have

$$\theta_j\alpha_i\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{(\sigma-1)\alpha_i\theta_i\theta_j}{\sigma} = \theta_i\alpha_j\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{(\sigma-1)\alpha_j\theta_j\theta_i}{\sigma}.$$

Taking sum over j,

$$\alpha_i\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{(\sigma-1)\alpha_i\theta_i}{\sigma} = \theta_i\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\theta_i\sum_{k=1}^{M}\frac{(\sigma-1)\alpha_k\theta_k}{\sigma},$$

$$\Leftrightarrow \alpha_i\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}-\frac{(\sigma-1)\theta_i}{\sigma}\right) = \theta_i\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}-\sum_{k=1}^{M}\frac{(\sigma-1)\alpha_k\theta_k}{\sigma}\right).$$

In the equation just above, on both sides, the terms in parentheses are strictly positive. Then, the following lemma can easily be shown.

Lemma A1

$\forall i \in \{1,...,M\}$,

$$\theta_i > \sum_{k=1}^{M}\alpha_k\theta_k \Leftrightarrow \alpha_i > \theta_i,$$

$$\theta_i = \sum_{k=1}^{M}\alpha_k\theta_k \Leftrightarrow \alpha_i = \theta_i,$$

$$\theta_i < \sum_{k=1}^{M}\alpha_k\theta_k \Leftrightarrow \alpha_i < \theta_i.$$

Now, if there exists i such that $\theta_i = \sum_{k=1}^{M}\alpha_k\theta_k$, let that i be the l. If not, take the minimum value of α_i's satisfying $\theta_i > \sum_{k=1}^{M}\alpha_k\theta_k$, and take one of the minimum, say α_o, and let o be the l. (2) is proved. Q.E.D.

Appendix 5

Proof of the existence and the uniqueness of θ:

The equation of θ (derived in Appendix 3b) is

$$\frac{\alpha_i}{\theta_i}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{(\sigma-1)\alpha_i}{\sigma} = \frac{\alpha_j}{\theta_j}\left(1-\sum_{k=1}^{M}\frac{\alpha_k\theta_k}{\sigma}\right)-\frac{(\sigma-1)\alpha_j}{\sigma}.$$

Here we specify the α and the summation in the equation as for the setting of the model.

$$\begin{aligned} \alpha_L &= \frac{1 - m\alpha_H}{M - m}, \\ \theta_L &= \frac{1 - m\theta_H}{M - m}, \\ \sum_{k=1}^{M} \alpha_k \theta_k &= m\alpha_H \theta_H + (M - m) \frac{1 - m\alpha_H}{M - m} \frac{1 - m\theta_H}{M - m}. \end{aligned}$$

Expanding them,

$$\begin{aligned} & \frac{\alpha_H}{\theta_H} \left[1 - \frac{m\alpha_H \theta_H + \frac{(1 - m\alpha_H)(1 - m\theta_H)}{M - m}}{\sigma} \right] - \frac{(\sigma - 1)\alpha_H}{\sigma} \\ = & \frac{1 - m\alpha_H}{1 - m\theta_H} \left[1 - \frac{m\alpha_H \theta_H + \frac{(1 - m\alpha_H)(1 - m\theta?H)}{M - m}}{\sigma} \right] - \frac{(\sigma - 1)(1 - m\alpha_H)}{\sigma(M - m)}. \end{aligned}$$

Multiplying $\theta_H(1 - m\theta_H)$ on both sides,

$$\begin{aligned} & \alpha_H(1 - m\theta_H) \left[1 - \frac{m\alpha_H \theta_H}{\sigma} - \frac{(1 - m\alpha_H)(1 - m\theta_H)}{\sigma(M - m)} \right] - \frac{(\sigma - 1)\alpha_H \theta_H (1 - m\theta)}{\sigma} \\ = & \theta_H(1 - m\alpha_H) \left[1 - \frac{m\alpha_H \theta_H}{\sigma} - \frac{(1 - m\alpha_H)(1 - m\theta_H)}{\sigma(M - m)} \right] - \frac{(\sigma - 1)(1 - m\alpha_H)\theta_H(1 - m\theta_H)}{\sigma(M - m)}. \end{aligned}$$

And eliminating the denominators,

$$\begin{aligned} & \alpha_H(1 - m\theta_H) \left[\sigma(M - m) - m(M - m)\alpha_H \theta_H - (1 - m\alpha_H)(1 - \theta_H) \right] - (\sigma - 1)(M - m)\alpha_H \theta_H (1 - m\theta_H) \\ = & \theta_H(1 - m\alpha_H) \left[\sigma(M - m) - m(M - m)\alpha_H \theta_H - (1 - m\alpha_H)(1 - \theta_H) \right] - (\sigma - 1)(1 - m\alpha_H)\theta_H(1 - m\theta_H). \end{aligned}$$

We can modify this into a kind of equational form:

$$(\alpha_H - \theta_H) \left[\sigma(M - m) - m(M - m)\alpha_H \theta_H - (1 - m\alpha_H)(1 - m\theta_H) \right] - (\sigma - 1)(M\alpha_H - 1)\theta_H(1 - m\theta_H) = 0.$$

Regarding this as the quadratic equation of theta,

$$G(\theta_H) = (\sigma M m\alpha_H - \sigma m)\theta^2 - (\sigma M - \sigma m - \sigma + (\sigma - 1)M\alpha_H + Mm\alpha^2)\theta + (\sigma M - \sigma m - 1)\alpha_H + m\alpha^2 = 0.$$

Under the setting of $M \geq m + 1$, $m \geq 1$, $\frac{1}{m} \geq \alpha \geq \frac{1}{M}$, we have

$$\begin{aligned} \sigma M m\alpha_H - \sigma m &\geq 0, \\ -[\sigma M - \sigma m - \sigma + (\sigma - 1)M\alpha_H + Mm\alpha_H^2] &< 0, \\ (\sigma M - \sigma m - 1)\alpha_H + m\alpha_H^2 &> 0. \end{aligned}$$

When $\theta_H = 1$,

$$\begin{aligned} G(1) &= \sigma M m\alpha_H - \sigma m - (\sigma M - \sigma m - \sigma + (\sigma - 1)M\alpha_H + Mm\alpha_H^2) + (\sigma M - \sigma m - 1)\alpha_H + m\alpha_H^2 \\ &= (m - mM)\alpha_H^2 + [\sigma M m - (\sigma - 1)M + \sigma M - \sigma m - 1]\alpha_H - \sigma(M + 1). \end{aligned}$$

Taking derivative by α_H,

$$\begin{aligned}\frac{d}{d\alpha_H}G(1) &= 2\alpha_H(m - Mm) + [\sigma Mm - (\sigma - 1)M + \sigma M - \sigma m - 1] = 0, \\ &\Rightarrow \quad \alpha_H = \frac{\sigma Mm + M - \sigma m - 1}{2m(M-1)} = \frac{\sigma m + 1}{2m} > 1 \quad (\because \sigma \geq 2).\end{aligned}$$

Since $m - mM < 0$, the first derivative above increasing with $\alpha_H < 1$. When $\alpha_H = \frac{1}{m}$,

$$\begin{aligned}&(m - mM)\alpha_H^2 + [\sigma Mm - (\sigma - 1)M + \sigma M - \sigma m - 1]\alpha_H - \sigma(M+1) \\ = \quad &\frac{1-M}{m} + \sigma M + \frac{M}{m} - \sigma - \frac{1}{m} - \sigma M - \sigma = -2\sigma < 0.\end{aligned}$$

Hence when $\theta_H = 1$, $G(\theta_H) < 0$. And $G(0) = (\sigma M - \sigma m - 1)\alpha_H + m\alpha_H^2 > 0$, therefore there exists unique solution of θ, which is:

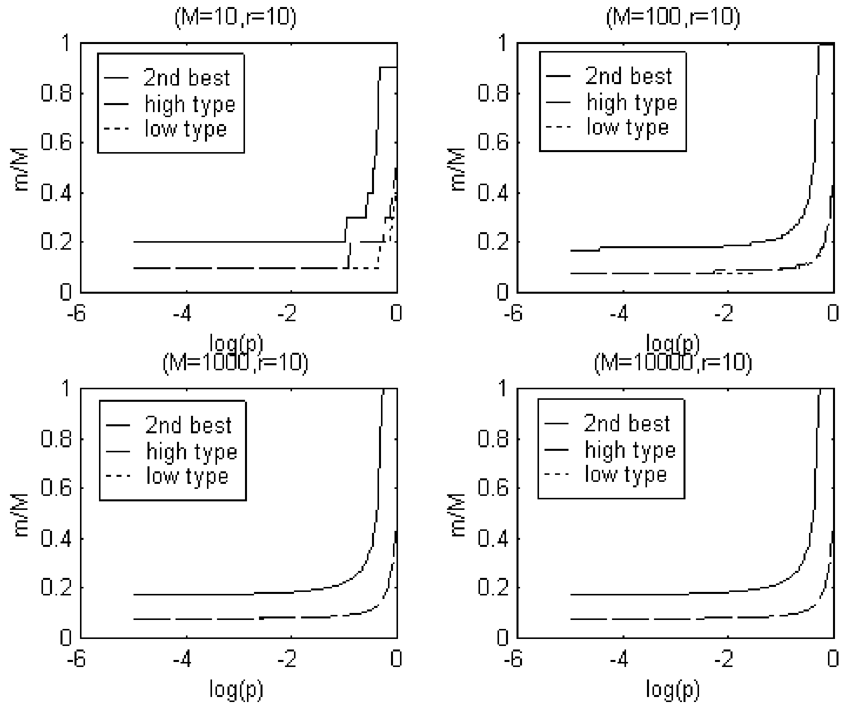
$$\theta_H = \frac{B - \sqrt{D}}{A},$$

where
$$\begin{aligned}&A = 2\sigma Mm\alpha_H - 2\sigma m, \\ &B = \sigma M - \sigma m - \sigma + M\alpha_H + Mm\alpha_H^2, \\ &D = (\sigma M - \sigma m - \sigma + M\alpha_H + Mm\alpha_H^2)^2 \\ &\qquad -4(\sigma Mm\alpha_H - \sigma m)[(\sigma M - \sigma m - 1)\alpha_H + m\alpha_H^2], \\ &\theta_L = \frac{1 - m\theta_H}{M - m},\end{aligned}$$

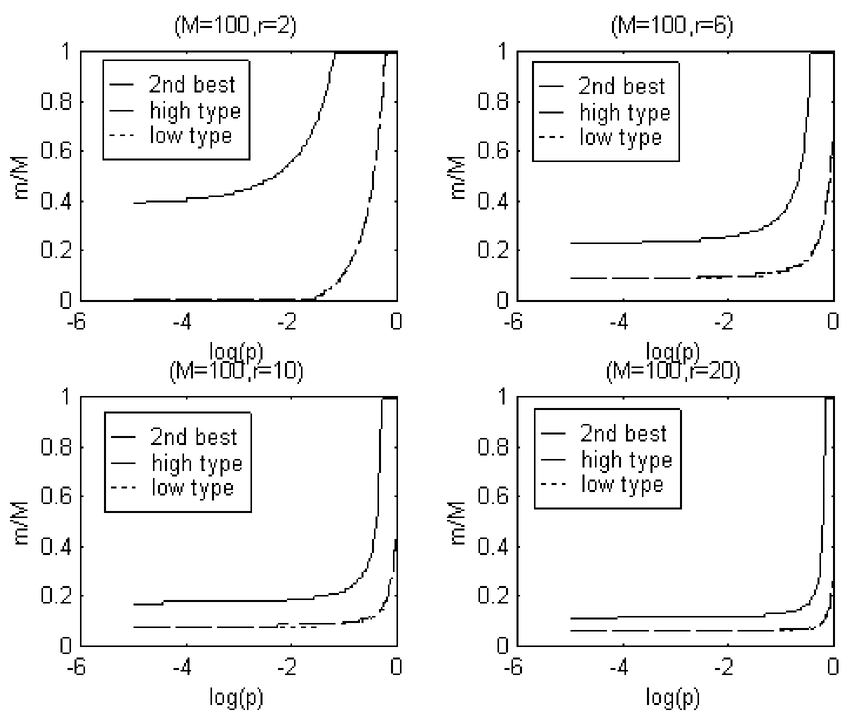
m: the number of the members with type of a_H.

Appendix 6

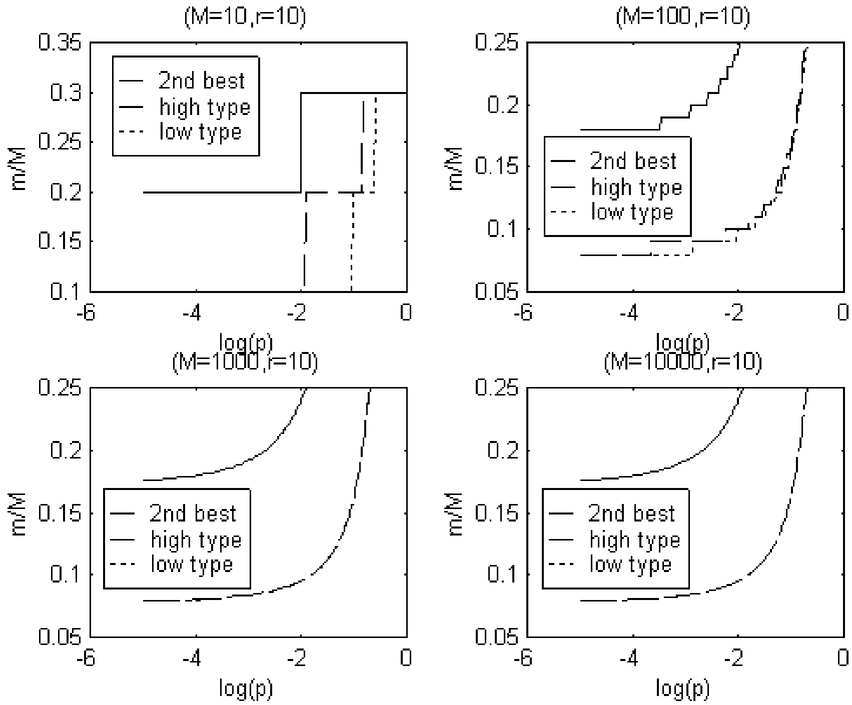
Case1, with r fixed to 10



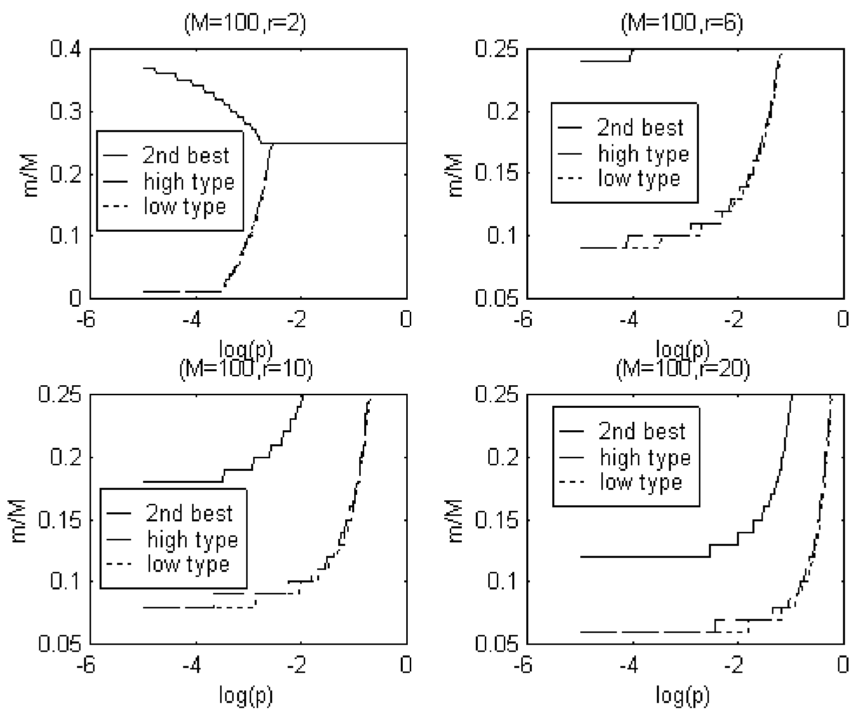
Case1, with M fixed to 100



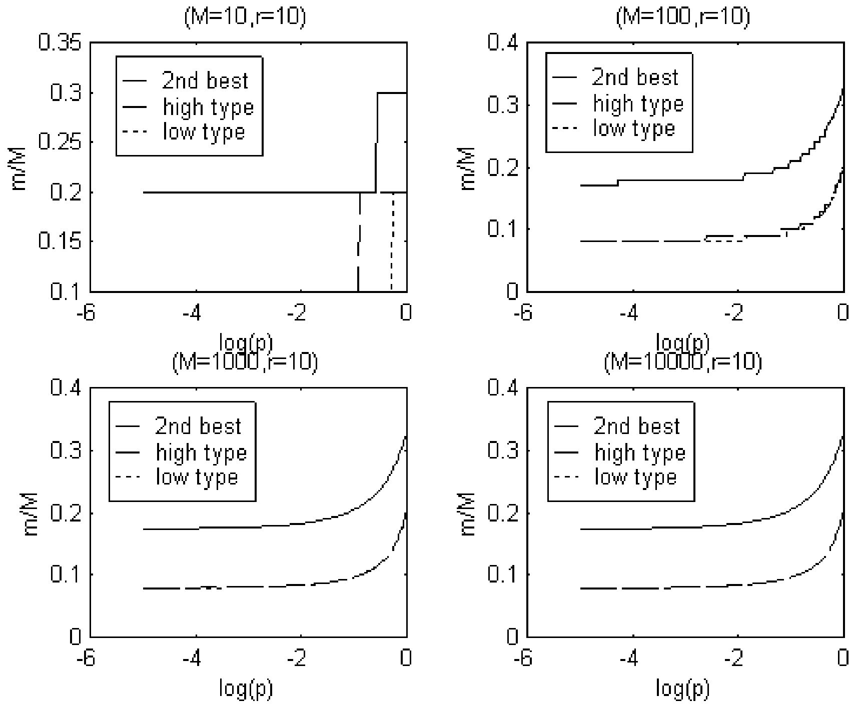
Case2, with r fixed to 10



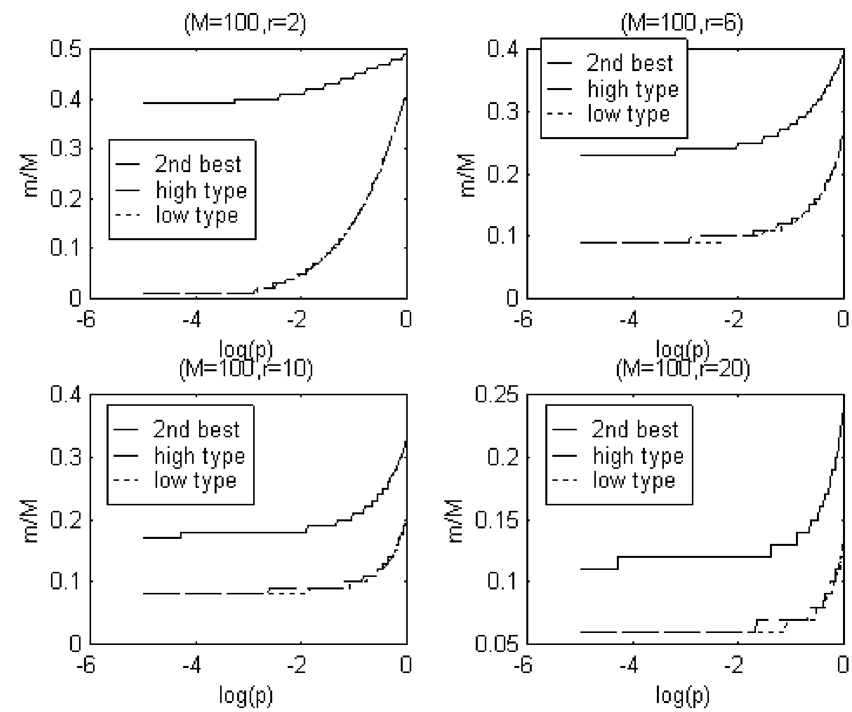
Case2, with M fixed to 100



Case3, with r fixed to 10



Case3, with M fixed to 100



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